(WC-)4DVar: learning priors and sovlers

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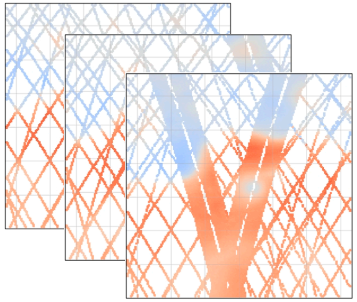
Melody meeting, October 2020

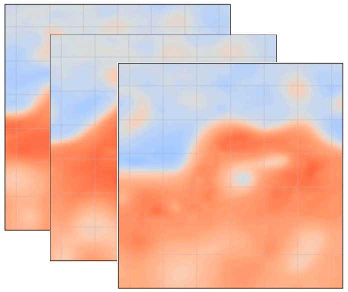
Lab-STICC

IMT Atlantique
Bretagne-Pays de la Loire
École Mines-Télécom

End-to-end learning for inverse problems (Fablet etal., 2020)



Partial observations y



True states x

Model-driven schemes:

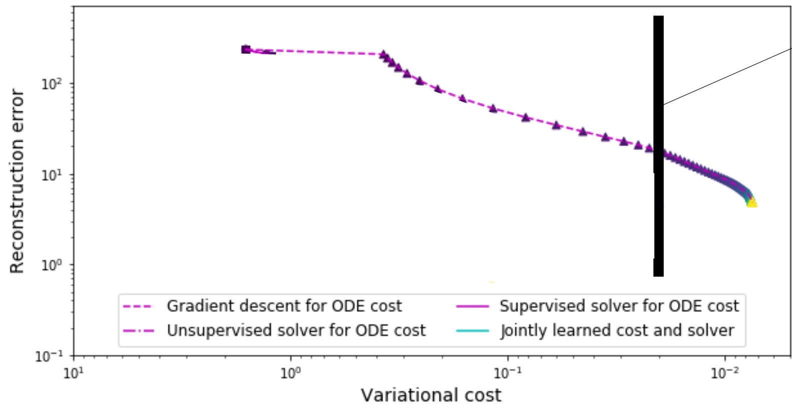
$$\arg\min_x \underbrace{\lambda_1 \|x-y\|_\Omega^2 + \lambda_2 \|x-\Phi(x)\|^2}_{U_\Phi\left(x^{(k)},y,\Omega\right)}$$

Gradient-based solver (adjoint/Euler-Lagrange method):

$$x^{(k+1)} = x^{(k)} - \alpha \nabla_x U_\Phi\left(x^{(k)},y,\Omega\right)$$

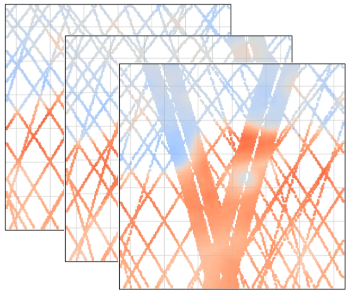
No control on the reconstruction error

$$x^{true} \neq \arg\min_x U_\Phi\left(x^{(k)},y,\Omega\right)$$

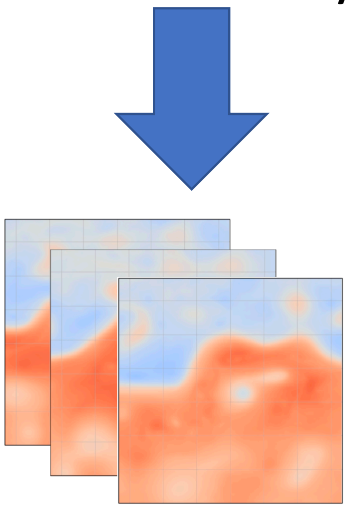


Variational cost
for the true state

End-to-end learning for inverse problems (Fablet et al., 2020)



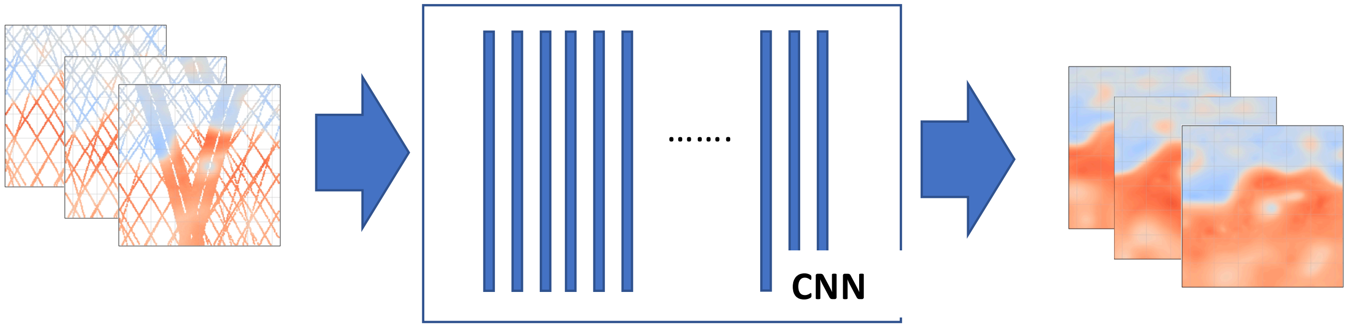
Partial observations y

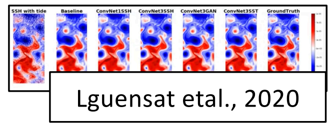


True states x

Model-driven schemes: $\widehat{x} = \arg\min_{x} \lambda_1 \left\| x - y \right\|_{\Omega}^{2} + \lambda_2 \Phi(x)$

Direct learning for inverse problems: $\widehat{x} = \Psi(y)$ $\quad y \rightarrow$ **CNN** $\rightarrow x$

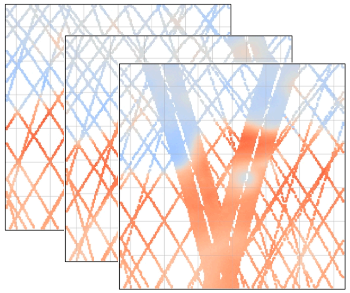




Examples of CNN architectures: Reaction-Diffusion architectures, ADMM-inspired architectures,...

Good performance but possibly weak interpretability/generaiization capacities of the solution byeond the training cases

End-to-end learning for inverse problems (Fablet et al., 2020)



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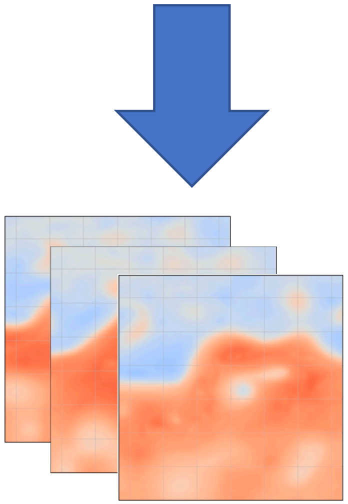


True states x

Model-driven schemes: $\arg\min_{x} \lambda_1 \|x - y\|_{\Omega}^2 + \lambda_2 \|x - \Phi(x)\|^2$

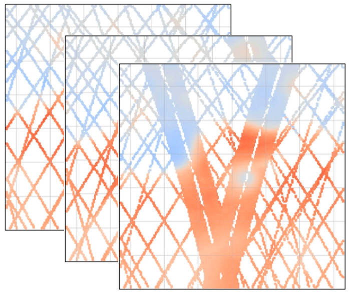
Direct learning for inverse problems: $\widehat{x} = \Psi(y)$ $\quad y \rightarrow \boxed{\textbf{CNN}} \rightarrow x$

Proposed scheme: joint learning of the variational model and solver

- **Theoretical bi-level optimization**

$$\arg\min_{\Phi} \sum_{n} \|x_n - \tilde{x}_n\|^2 \text{ s.t. } \tilde{x}_n = \arg\min_{x_n} U_{\Phi}(x_n, y_n, \Omega_n)$$

End-to-end learning for inverse problems (Fablet et al., 2020)



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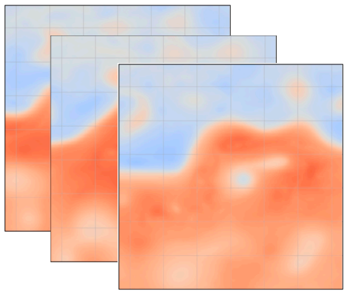

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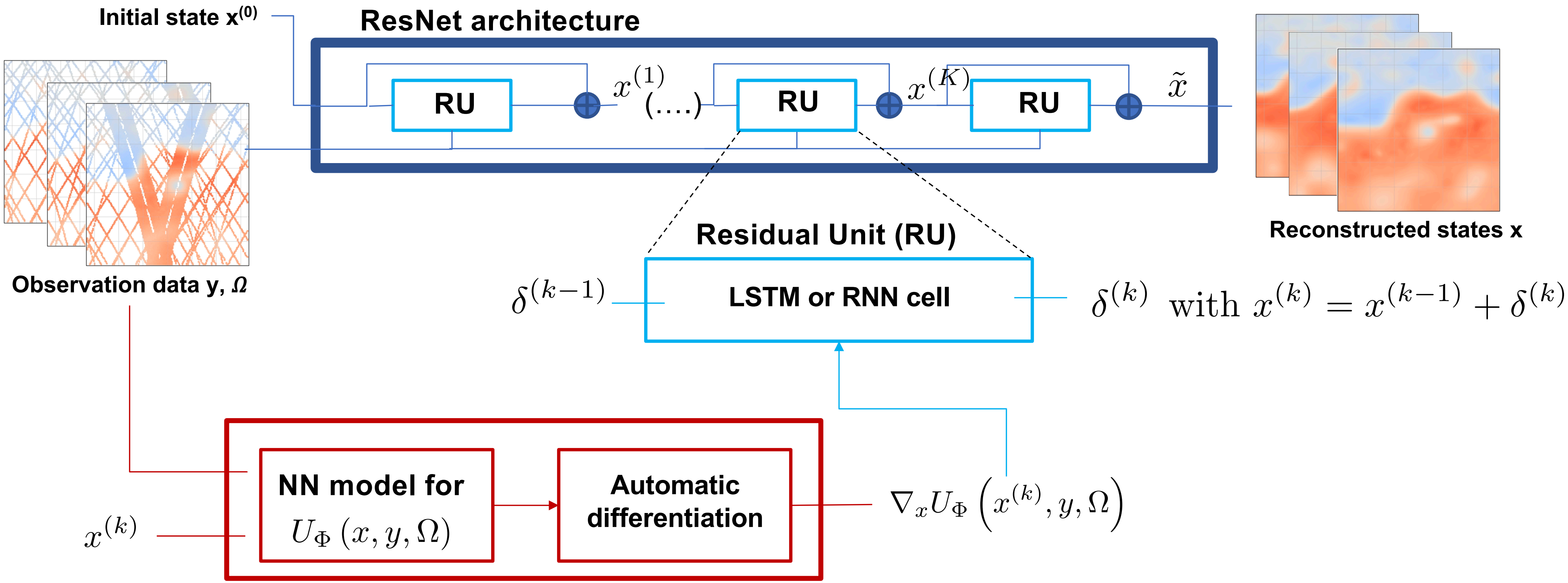
- **Restated with a gradient-based NN solver for inner minimization**

$$\arg\min_{\Phi,\Gamma} \sum_n \|x_n - \tilde{x}_n\|^2 \text{ s.t. } \tilde{x}_n = \Psi_{\Phi,\Gamma}(x_n^{(0)}, y_n, \Omega_n)$$

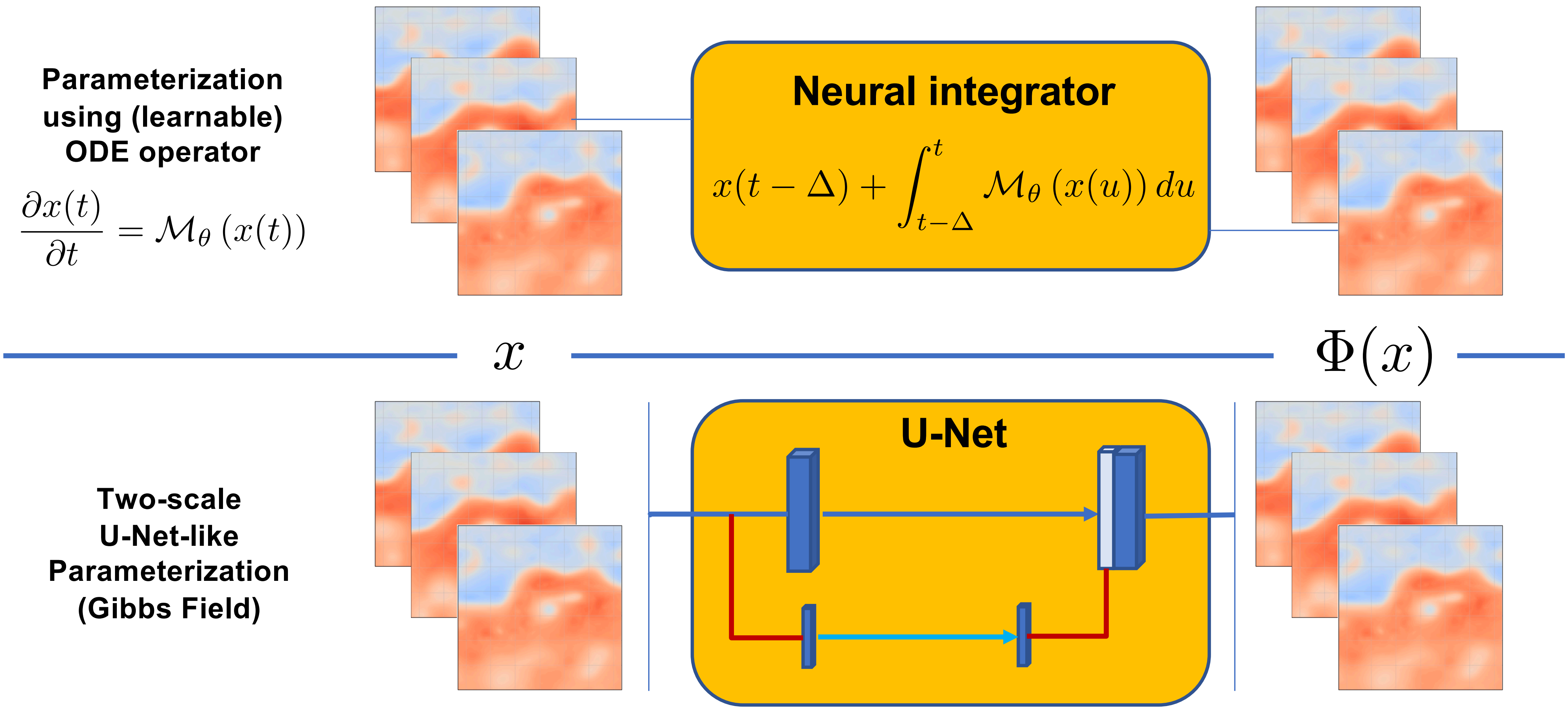
Iterative NN solver using automatic differentiation to compute gradient $\nabla_x U_\Phi\left(x^{(k)}, y, \Omega\right)$

End-to-end learning for inverse problems (Fablet et al., 2020)

Proposed scheme: associated NN architecture

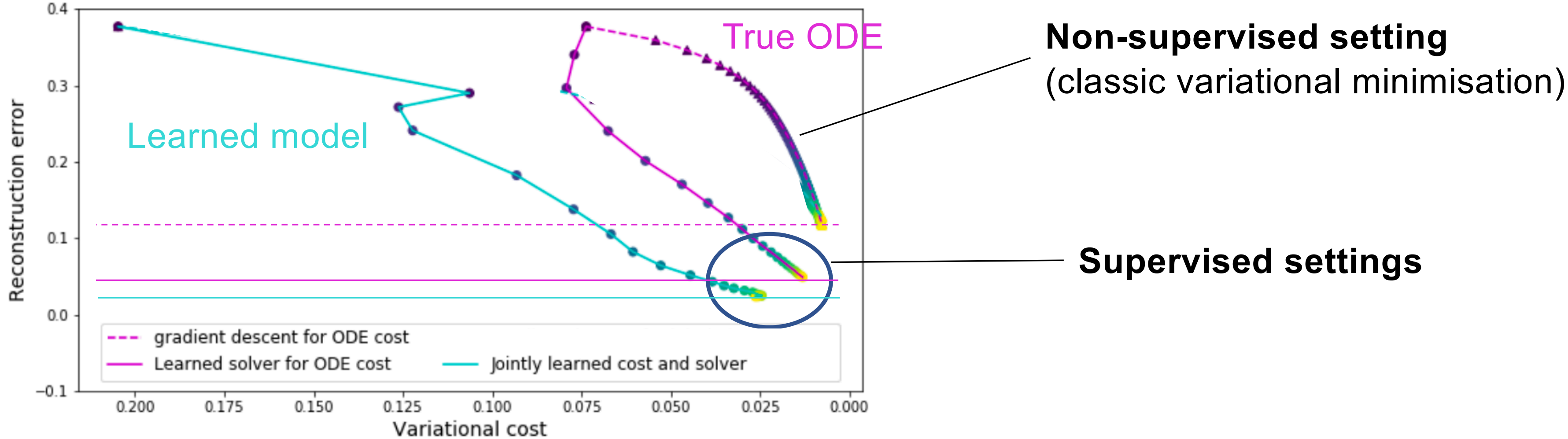


End-to-end learning for 4DVar DA: projection operator Φ

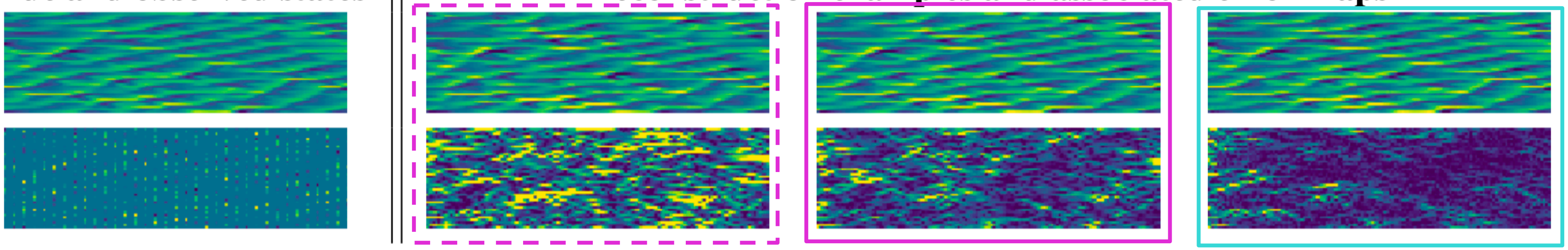


End-to-end learning for inverse problems (Fablet etal., 2020)

Illustration on Lorenz-96 dynamics (Bilinear ODE)



True and observed states || **Reconstruction examples and associated error maps**



End-to-end learning for inverse problems (Fablet etal., 2020)

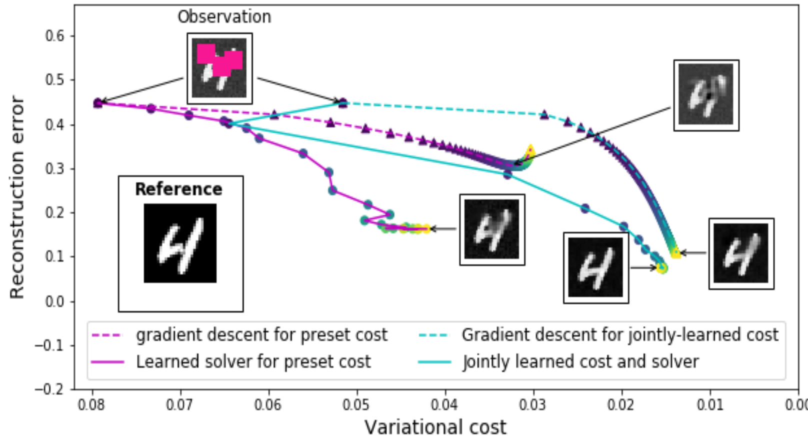
Key messages

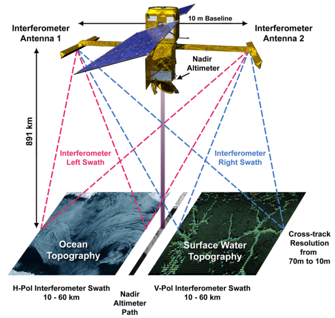
- **We can bridge DNN and variational models to solve inverse problems**

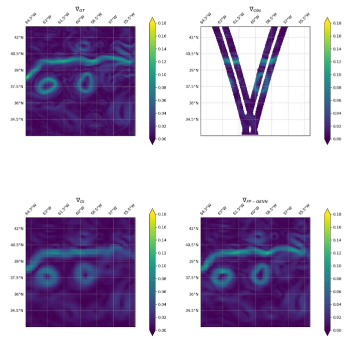
- **Learning both variational priors and solvers using groundtruthed (simulation) or observation-only data**

- **The best model may not be the TRUE one for inverse problems**

- **Generic formulation/architecture beyond space-time dynamics** (ongoing application to short-term forecasting)







Preprint: https://arxiv.org/abs/2006.03653

Code: https://github.com/CIA-Oceanix

Thank you.

Joint work with M. Beauchamp, B. Chapron, L. Drumetz, E. Mémin, O. Pannekoucke, F. Rousseau

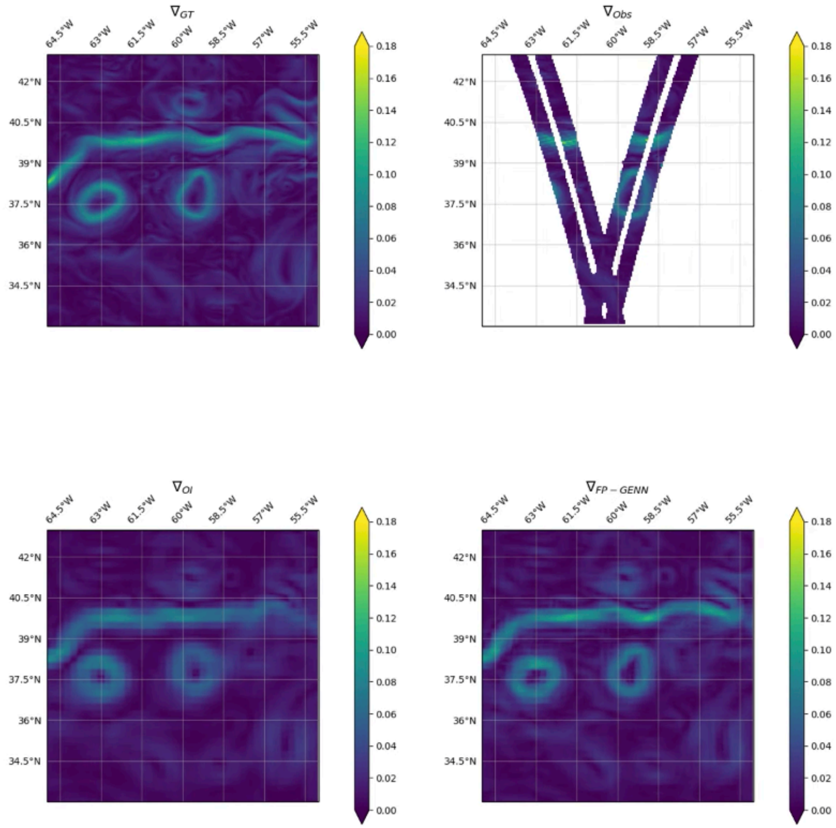
More:

- **Webpage:** https://rfablet.github.io/
- **Preprints:** https://www.researchgate.net/profile/Ronan_Fablet

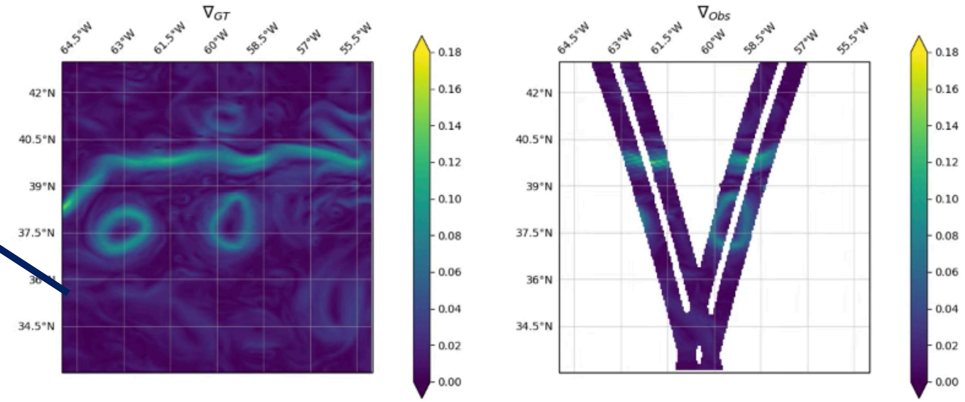
End-to-end learning for inverse problems (Fablet etal., 2020)

Applications to the reconstruction of sea surface current from SWOT data

NB: preliminary results with a fixed-point
Solver rather than a gradient-based solver

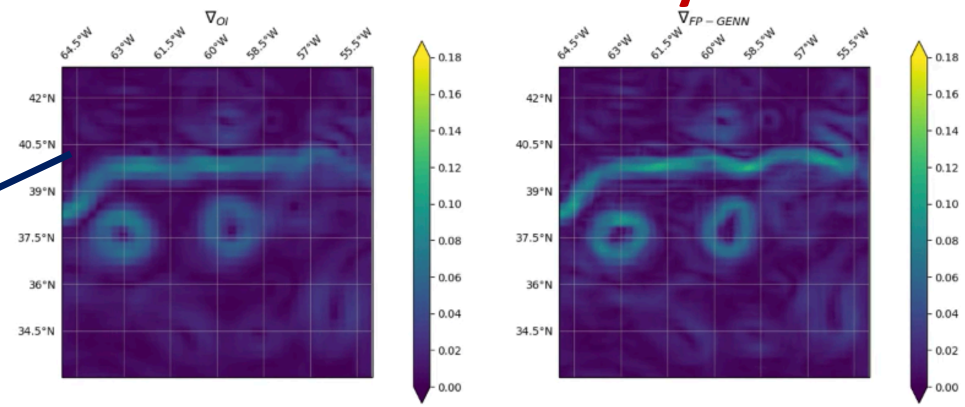


An example for upcoming SWOT mission



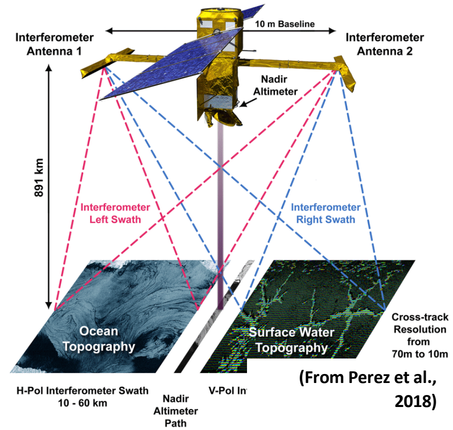
Groundtruth

Can we learn how to best reconstruct surface dynamics from satellite data ? Can we directly learn observation data?



State-of-the-art operational processing

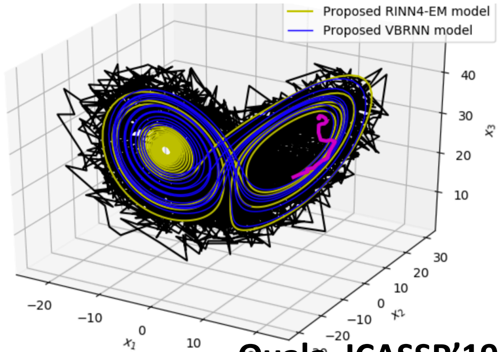
Proposed NN framework (Fablet et al., 2019)



(From Perez et al., 2018)

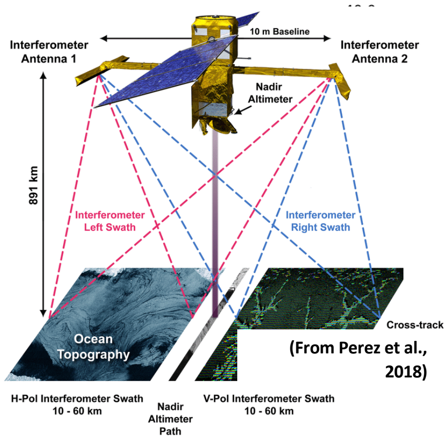
End-to-end learning from real observation data

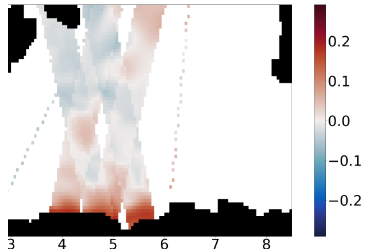
Scarce time sampling



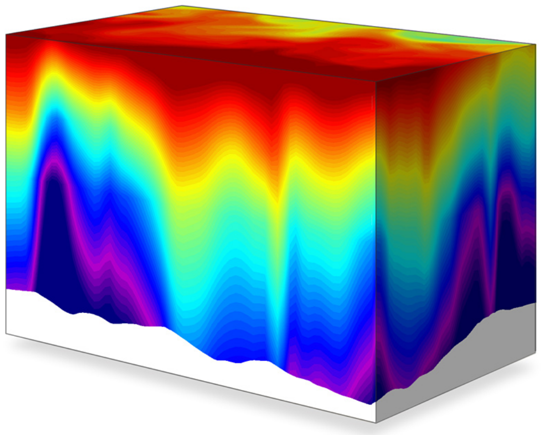
Ouala, ICASSP'19

Noisy and irregular sampling





Nguyen, ICASSP'20



Partially-observed system

Ouala, preprint 2019